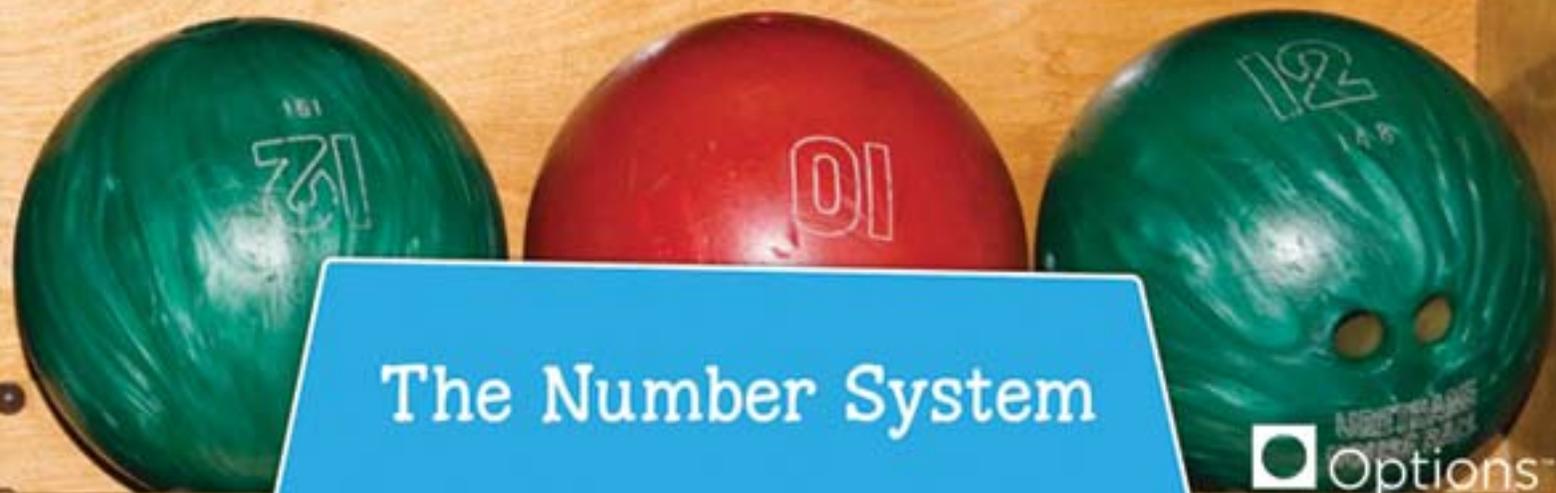


# COMMON CORE

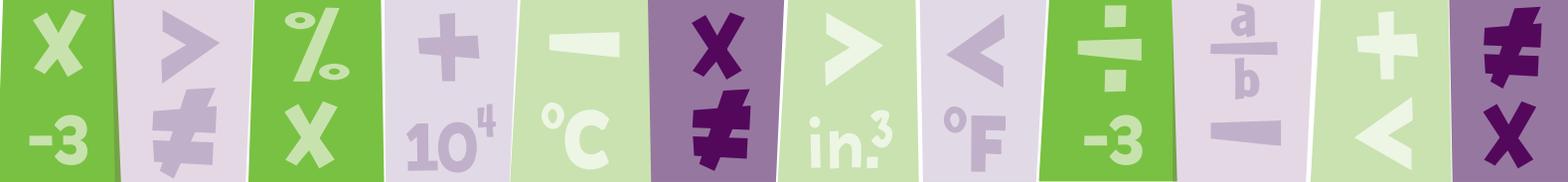
Grade 6

Mathematics

# CLINICS



The Number System



**Module**  
**1**

# The Number System

**Common Core State Standards**

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<b>Lesson 2</b>	Divide Whole Numbers . . . . .	8	6.NS.2
<b>Lesson 3</b>	Integers . . . . .	12	6.NS.5, 6.NS.6.a, 6.NS.6.c
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<b>Lesson 7</b>	Add and Subtract Decimals. . . . .	28	6.NS.3
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# Factors and Multiples

## Key Words

factor  
 greatest common factor  
 least common multiple  
 multiple  
 whole number

A **whole number** is in the set of counting numbers and zero:  $\{0, 1, 2, 3, 4, \dots\}$ .

A **multiple** of a number is the product of the number and any whole number. The **least common multiple**, or **LCM**, is the smallest number that is a multiple of two numbers. Zero is not considered a common multiple of two numbers.

A **factor** is a number that divides evenly into another number. Every whole number greater than 1 has at least 2 factors: itself and 1. The **greatest common factor**, or **GCF**, is the largest number that is a factor of two numbers.

## Example 1

What is the least common multiple (LCM) of 8 and 12?

List the first 10 multiples of 8 and 12. Look for common multiples.

multiples of 8: 0, 8, 16, **24**, 32, 40, **48**, 56, 64, **72**, ...

multiples of 12: 0, 12, **24**, 36, **48**, 60, **72**, 84, 96, 108, ...

There are three common multiples in the lists: 24, 48, and 72. The smallest number out of those three multiples is 24.

The least common multiple of 8 and 12 is 24.

## Example 2

What is the greatest common factor (GCF) of 12 and 20?

List the factors of 12 and 20. Look for common factors.

factors of 12: **1**, **2**, 3, **4**, 6, and 12.

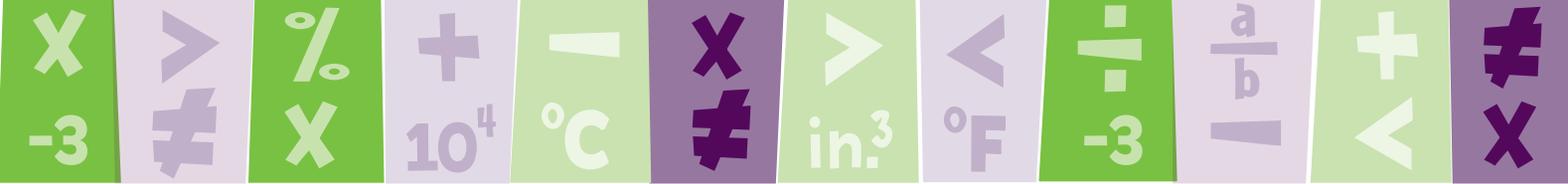
factors of 20: **1**, **2**, **4**, 5, 10, and 20.

There are three common factors in the lists: 1, 2, and 4. The greatest number out of those three factors is 4.

The greatest common factor of 12 and 20 is 4.

## IDENTIFY

What are the LCM and GCF of 4 and 6?



## Guided Practice

1 What is the least common multiple of 6 and 10?

**Step 1** List the first 10 multiples of 6.

\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_,  
\_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_

**Step 2** List the first 10 multiples of 10.

\_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_

**Step 3** Find the common multiples in the lists.  
Then find the smallest number out of  
those common multiples.

The smallest number is \_\_\_\_\_.

The least common multiple of 6 and 10 is \_\_\_\_\_.

### REMEMBER

The first multiple of a whole number is 0, but 0 cannot be LCM.

### THINK

There may be more than one common multiple in your lists. Find the *least* common multiple.

2 What is the greatest common factor of 16 and 26?

**Step 1** List all the factors of 16.

\_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_

**Step 2** List all the factors of 26.

\_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_

**Step 3** Find the common factors in the lists. Then find the  
greatest number out of those common factors.

The greatest number is \_\_\_\_\_.

The greatest common factor of 16 and 26 is \_\_\_\_\_.

### REMEMBER

The first factor of any whole number is 1.

## Independent Practice

1. How can you find the greatest common factor of two numbers?

---



---

2. What is a multiple of a number?

---



---

**Ask Yourself**

What are the multiples of each number?

What are the factors of each number?

Find the least common multiple of each pair of numbers.

3. 4 and 5 \_\_\_\_\_

4. 2 and 5 \_\_\_\_\_

5. 1 and 7 \_\_\_\_\_

6. 3 and 9 \_\_\_\_\_

7. 4 and 8 \_\_\_\_\_

8. 4 and 6 \_\_\_\_\_

Find the greatest common factor of each pair of numbers.

9. 3 and 4 \_\_\_\_\_

10. 2 and 8 \_\_\_\_\_

11. 5 and 10 \_\_\_\_\_

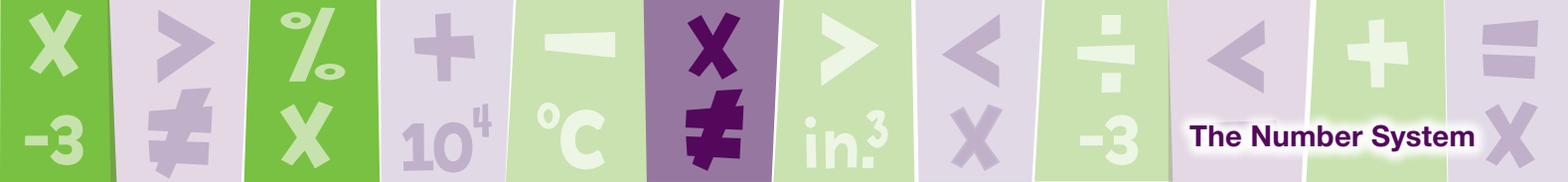
12. 8 and 12 \_\_\_\_\_

13. 10 and 15 \_\_\_\_\_

14. 9 and 15 \_\_\_\_\_

15. Hot dogs come in packs of 6. Buns come in packs of 8. What is the fewest number of hot dogs or buns you can buy to have the same number of each?

---



Find the least common multiple of each pair of numbers.

16. 2 and 11 \_\_\_\_\_

17. 6 and 12 \_\_\_\_\_

18. 9 and 11 \_\_\_\_\_

19. 7 and 10 \_\_\_\_\_

20. 5 and 7 \_\_\_\_\_

21. 8 and 12 \_\_\_\_\_

22. 10 and 11 \_\_\_\_\_

23. 7 and 9 \_\_\_\_\_

Find the greatest common factor of each pair of numbers.

24. 75 and 20 \_\_\_\_\_

25. 80 and 90 \_\_\_\_\_

26. 14 and 49 \_\_\_\_\_

27. 27 and 63 \_\_\_\_\_

28. 38 and 95 \_\_\_\_\_

29. 17 and 37 \_\_\_\_\_

30. 22 and 90 \_\_\_\_\_

31. 39 and 78 \_\_\_\_\_

Solve each problem.

32. Alfonso buys 12 bagels and 8 muffins. He wants to split the items into bags with the same number of bagels and the same number of muffins in each bag. What is the greatest number of bags that Alfonso can make without any items left over?

\_\_\_\_\_

33. There are 24 students in a 6th-grade class. There are 30 students in a 7th-grade class. The students in each class are split into equal-size groups for a field trip. What is the greatest number of groups to make if there is the same number of 6th-grade students and the same number of 7th-grade students in each group?

\_\_\_\_\_

# COMMON CORE

Grade 6

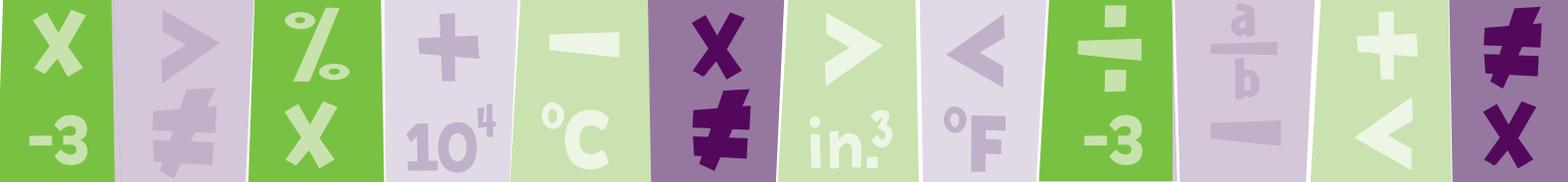
Mathematics

# CLINICS



Ratios/Proportional Relationships  
and Expressions/Equations

Options™



**Module**  
**2**

# Ratios and Proportional Relationships; Expressions and Equations

**Common Core State Standards**

<b>Lesson 1</b>	Ratios . . . . .	4	6.RP.1
<b>Lesson 2</b>	Equivalent Ratios . . . . .	8	6.RP.3.a
<b>Lesson 3</b>	Unit Rates . . . . .	12	6.RP.2, 6.RP.3.b
<b>Lesson 4</b>	Percents. . . . .	16	6.RP.3.c
<b>Lesson 5</b>	Convert Measurements . . . . .	20	6.RP.3.d
<b>Lesson 6</b>	Write Expressions . . . . .	24	6.EE.1, 6.EE.2.a, 6.EE.2.b, 6.EE.6
<b>Lesson 7</b>	Evaluate Expressions . . . . .	28	6.EE.1, 6.EE.2.c
<b>Lesson 8</b>	Equivalent Expressions . . . . .	32	6.EE.3, 6.EE.4
<b>Lesson 9</b>	Solve Equations. . . . .	36	6.EE.5, 6.EE.7
<b>Lesson 10</b>	Linear Equations . . . . .	40	6.EE.9
<b>Lesson 11</b>	Use Equations to Solve Problems . . . . .	44	6.EE.6, 6.EE.7, 6.EE.9
<b>Lesson 12</b>	Inequalities . . . . .	48	6.EE.5, 6.EE.6, 6.EE.8
<b>Glossary</b>	. . . . .	52	
<b>Math Tools</b>	. . . . .	53	



# Ratios

## Key Words

denominator  
greatest common factor (GCF)  
numerator  
ratio

A **ratio** is a comparison between two numbers. A ratio can be written in three ways.

3 to 4

3:4

$\frac{3}{4}$

The order of the numbers in a ratio is important. The first number being compared comes first in the ratio. When a ratio is expressed as a fraction, the first number appears as the **numerator** and the second number appears as the **denominator**.

To simplify a ratio, divide both numbers by the **greatest common factor (GCF)**.

## Example 1

What is the ratio of squares to circles? Describe the ratio in words.

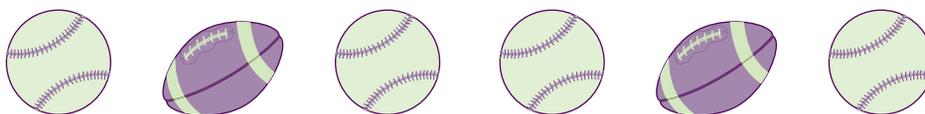


There are 4 squares. There are 3 circles.  
Describe the ratio with squares first.

The ratio of squares to circles is 4 to 3.

## Example 2

What is the ratio of footballs to all the balls?



There are 2 footballs. There are 6 balls in all.

The ratio of footballs to balls is 2 to 6 or  $\frac{2}{6}$ .

This can be simplified:  $\frac{2 \div 2}{6 \div 2} = \frac{1}{3}$ .

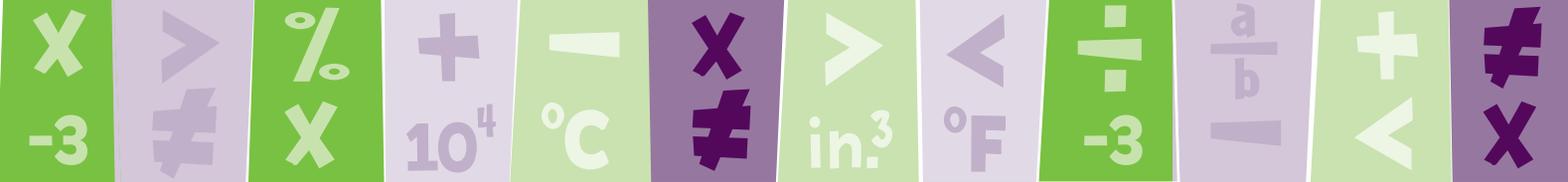
For every 1 football, there are 3 balls.

The ratio of footballs to all the balls is 1 to 3.

It can also be written as 1:3 or  $\frac{1}{3}$ .

## LIST

A fruit bowl has 3 apples and 1 banana.  
Write three different ratios describing the fruits in the bowl.



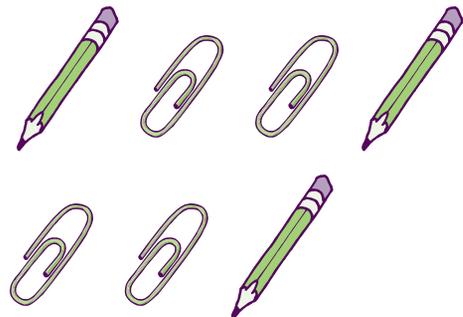
## Guided Practice

- 1 Write the ratio of pencils to paper clips as a fraction. Then, describe the ratio using words.

**Step 1** Count the number of pencils.  
Count the number of paper clips.

There are \_\_\_\_\_ pencils.

There are \_\_\_\_\_ paper clips.



**Step 2** Write a fraction comparing the pencils to the paper clips.

$$\frac{\text{pencils}}{\text{paper clips}} = \frac{\square}{\square}$$

The ratio of pencils to paper clips is  $\frac{\square}{\square}$ .

For every \_\_\_\_\_ pencils, there are \_\_\_\_\_ paper clips.

### REMEMBER

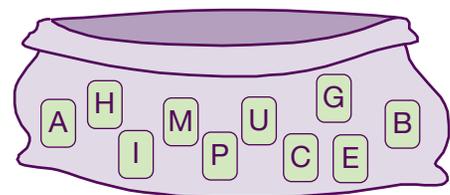
Write the first number being compared in a ratio as the numerator.

- 2 Write the ratio of vowels to all letters in the bag as  $a:b$ . Then, describe the ratio using words.

**Step 1** Count the number of vowels.  
Count the number of letters in total.

There are \_\_\_\_\_ vowels.

There are \_\_\_\_\_ letters in total.



**Step 2** Write a ratio comparing the vowels to the letters in total.  
Simplify by using the GCF.

vowels: all letters = \_\_\_\_\_ : \_\_\_\_\_

$4 \div \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$       $10 \div \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

### THINK

To find the greatest common factor, find the greatest whole number that divides evenly into 4 and 10.

The ratio of vowels to letters in the bag is \_\_\_\_\_.

For every \_\_\_\_\_ vowels, there are \_\_\_\_\_ letters in the bag.

# Independent Practice

1. How can you express a ratio as a fraction?

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2. How do you simplify a ratio?

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**Ask Yourself**

Which object is mentioned first in the ratio?

Which number should be listed first in the ratio?

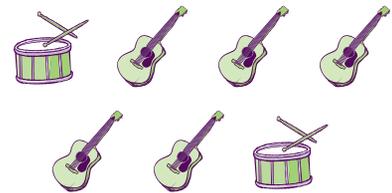
Write each ratio in three ways.

3. What is the ratio of moons to stars?



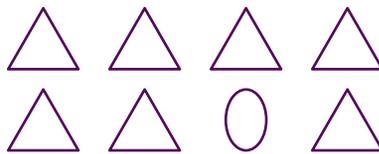

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4. What is the ratio of guitars to drums?




---

5. What is the ratio of triangles to ovals?




---

6. What is the ratio of males to people?




---

7. In one week, a computer shop sold 13 laptop computers and 7 desktop computers. What was the ratio of laptop computers to all computers that were sold in the week? Describe the ratio using words.

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# COMMON CORE

Grade 6

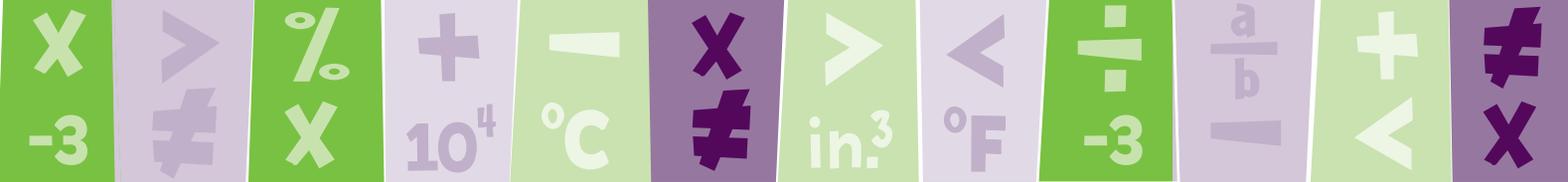
Mathematics

# CLINICS



Geometry

Options



**Module**  
**3**

# Geometry

**Common Core State Standards**

<b>Lesson 1</b>	Areas of Triangles . . . . .	4	6.G.1
<b>Lesson 2</b>	Areas of Quadrilaterals . . . . .	8	6.G.1
<b>Lesson 3</b>	Areas of Composite Polygons . . . . .	12	6.G.1
<b>Lesson 4</b>	Polygons on the Coordinate Plane . . . . .	16	6.G.3
<b>Lesson 5</b>	Solve Problems with Area. . . . .	20	6.G.1
<b>Lesson 6</b>	Solid Figures . . . . .	24	6.G.4
<b>Lesson 7</b>	Surface Area . . . . .	28	6.G.4
<b>Lesson 8</b>	Volume . . . . .	32	6.G.2
<b>Glossary</b>	. . . . .	36	
<b>Math Tools</b>	. . . . .	39	

# Areas of Triangles

## Key Words

area  
rectangle  
triangle

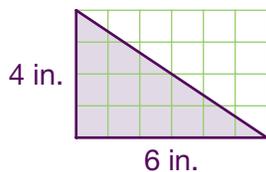
**Area** is the measure of the space inside a two-dimensional figure. Area ( $A$ ) is measured in square units, such as square meters ( $m^2$ ) or square feet ( $ft^2$ ).

Two **triangles** with the same shape and size can be used to form a **rectangle**. The area of the rectangle is the product of its length and width:  $A = lw$ . The area of one triangle, therefore, is half the area of the rectangle.

The formula for the area of a triangle is  $A = \frac{1}{2}bh$ , where  $b$  stands for the base and  $h$  stands for the height. The base of the triangle is related to the length of the rectangle. The height of the triangle is related to the width of the rectangle.

## Example

The rectangular grid has a length of 6 inches and a height of 4 inches. A shaded triangle is placed over the grid. What is the area of the shaded triangle?



Find the area of the rectangular grid.

$$A = lw$$

$$A = 6 \text{ in.} \times 4 \text{ in.} = 24 \text{ in.}^2$$

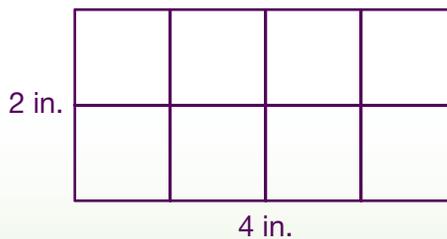
The area of the triangle is half the area of the grid.

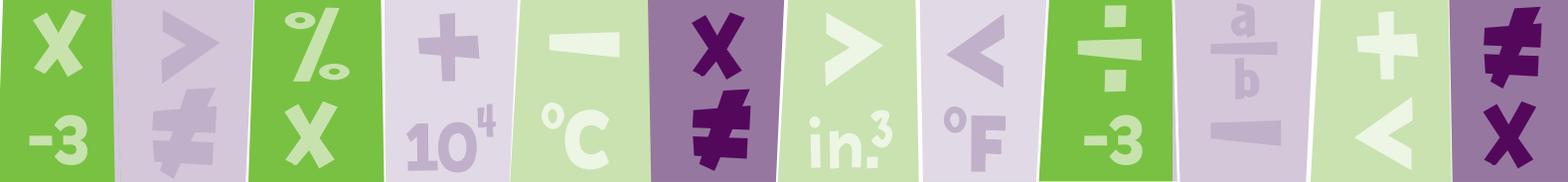
$$24 \text{ in.}^2 \div 2 = 12 \text{ in.}^2$$

The area of the shaded triangle is  $12 \text{ in.}^2$ .

## DRAW

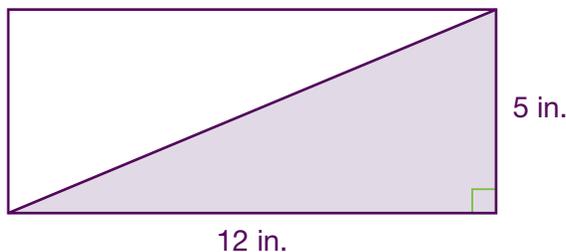
Draw a triangle with an area of  $4 \text{ in.}^2$  on the following grid.





## Guided Practice

- 1 A shaded triangle is connected to another triangle with the same dimensions to form a rectangle. What is the area of the shaded triangle?



**Step 1** Determine the area of the rectangle.

$$A = \underline{\quad} \times \underline{\quad}$$

$$A = \underline{\quad}$$

### THINK

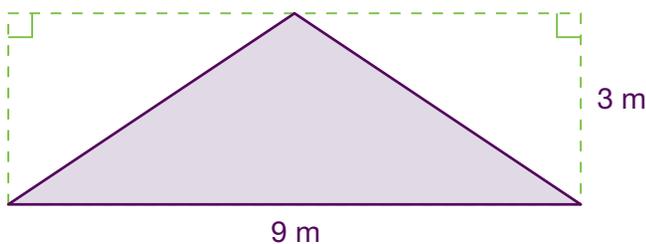
The commutative property of multiplication states that the order of the factors does not matter. So the order of the length and the width does not matter.

**Step 2** Divide the area of the rectangle by 2 to find the area of the triangle.

$$A = \underline{\quad} \div 2 = \underline{\quad}$$

The area of the shaded triangle is \_\_\_\_\_.

- 2 What is the area of the shaded triangle?



**Step 1** Write the formula for the area of a triangle.

$$A = \frac{1}{2} \times \underline{\quad} \times \underline{\quad}$$

**Step 2** Substitute the values into the formula. Then solve.

$$A = \frac{1}{2} \times \underline{\quad} \times \underline{\quad}$$

$$A = \frac{1}{2} \times \underline{\quad}$$

$$A = \underline{\quad}$$

### REMEMBER

The length of the rectangle is the base of the triangle.  
The width of the rectangle is the height of the triangle.

The area of the shaded triangle is \_\_\_\_\_.

# Independent Practice

1. What is area?

---



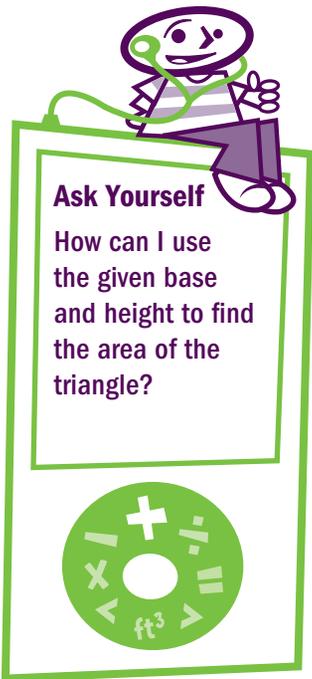
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2. How can you find the area of any triangle?

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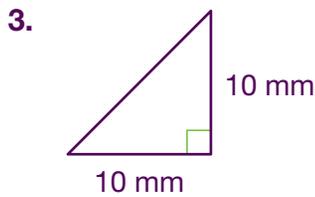


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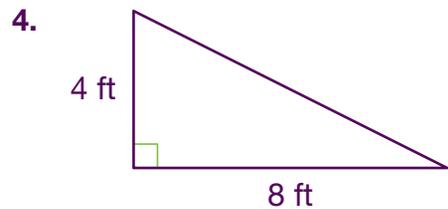


**Ask Yourself**  
 How can I use the given base and height to find the area of the triangle?

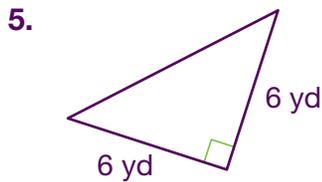
Find the area of each triangle.



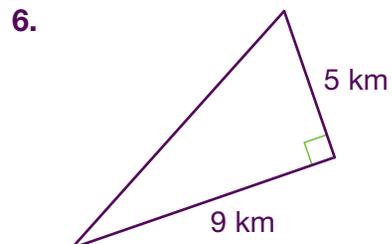

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**Solve.**

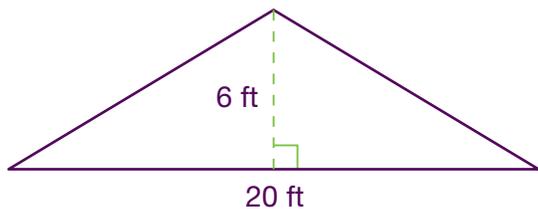
7. The sail of a sailboat is in the shape of a right triangle. Its height is 6 ft and its base is 4 ft. What is the area of the sailboat's sail?

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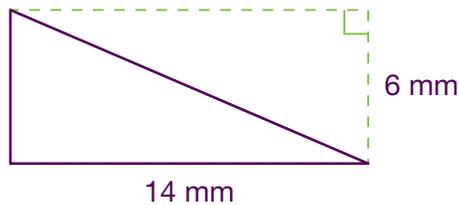
Find the area of each triangle.

8.



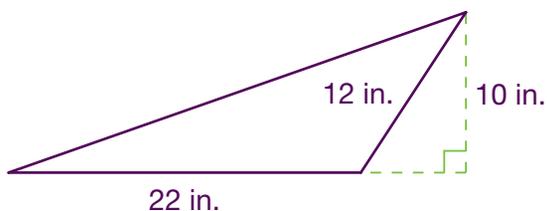
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9.



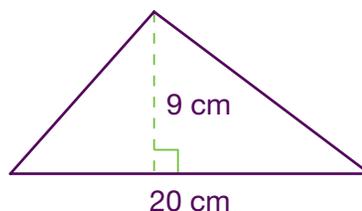
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10.



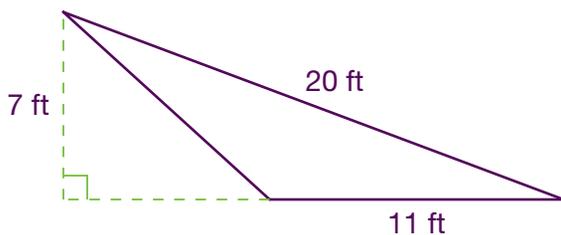
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11.



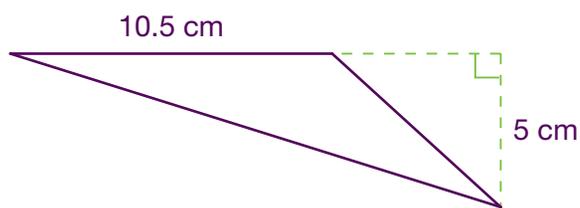
\_\_\_\_\_

12.



\_\_\_\_\_

13.



\_\_\_\_\_

Solve each problem.

14. A farmer walks along the perimeter of her triangular field by walking 13.5 yards north, walking 24.4 yards west, and then returning to the original spot. What is the area of the farmer's field?

\_\_\_\_\_

15. A jeweler carves a piece of driftwood into a flat triangular shape to create a pendant. The base of the driftwood is 28.5 mm, and its height is 12 mm. What is the area of the jeweler's piece of driftwood?

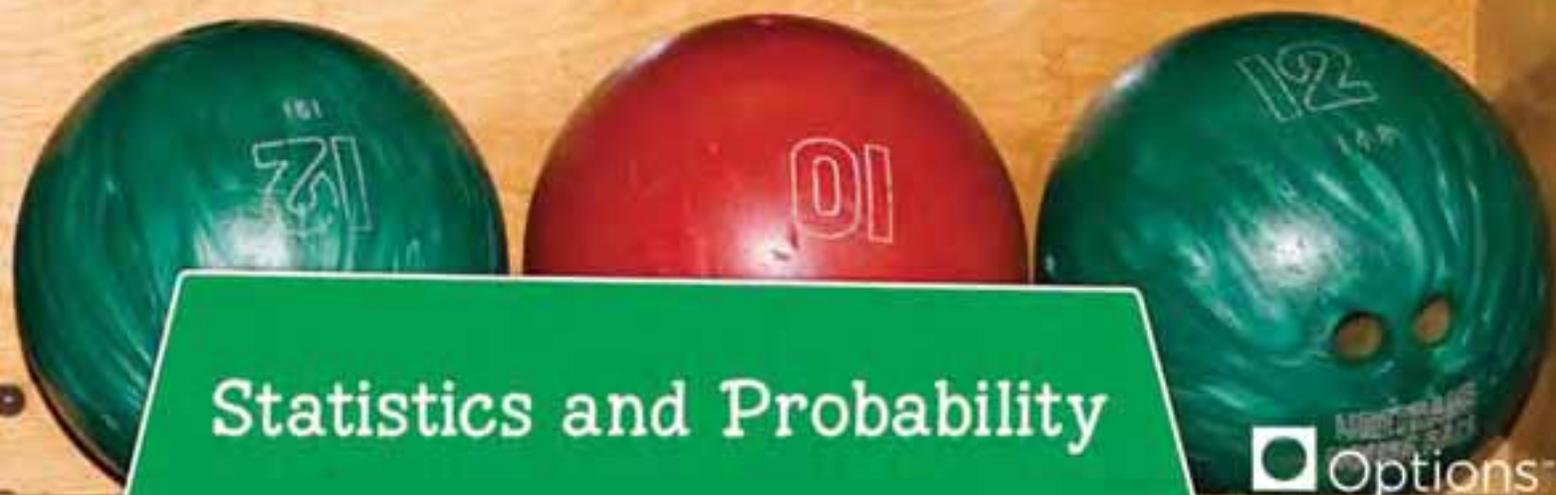
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# COMMON CORE

Grade 6

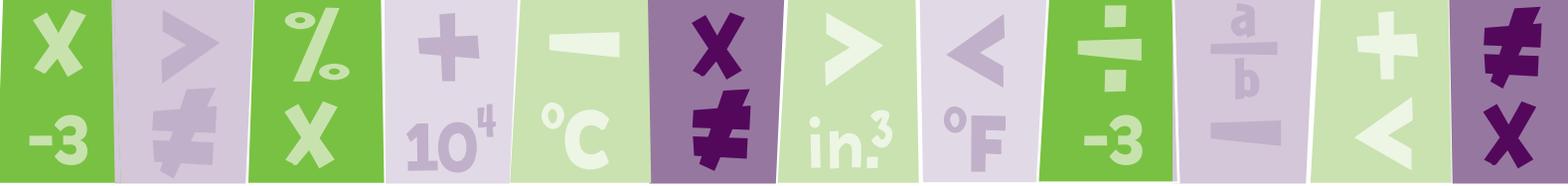
Mathematics

# CLINICS



Statistics and Probability

Options



**Module**  
**4**

# Statistics and Probability

**Common Core State Standards**

<b>Lesson 1</b>	Measures of Center . . . . .	4	6.SP.1, 6.SP.2, 6.SP.3
<b>Lesson 2</b>	Measures of Variability . . . . .	8	6.SP.2, 6.SP.3
<b>Lesson 3</b>	Dot Plots . . . . .	12	6.SP.4, 6.SP.5.a, 6.SP.5.c
<b>Lesson 4</b>	Use a Plot to Choose the Best Measure . . . . .	16	6.SP.5.d
<b>Lesson 5</b>	Box Plots . . . . .	20	6.SP.4, 6.SP.5.b, 6.SP.5.c
<b>Lesson 6</b>	Histograms . . . . .	24	6.SP.4, 6.SP.5.a, 6.SP.5.c
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# Measures of Center

## Key Words

data  
mean  
measure of  
center  
median  
mode

**Data** is a collection of information, such as costs, ages, or weights.

A **measure of center** is a measurement that summarizes a data set with a single number. Measures of center include the mean, median, and mode.

The **mean** is the sum of the values in a data set divided by the number of values in the set. The **median** is the middle value in a data set when it is in numerical order. The **mode** is the value that appears most often in a data set.

A question is statistical when the data collected to answer the question contains variability. A question is not statistical if the data does not contain variability. For example, the question, “What are the heights of the last 10 U.S. presidents?” is a statistical question. The answers, or the data set, will vary since the presidents are not all the same height.

The data for a statistical question can be described by its measures of center.

## Example

The ages of the 10 friends at a party are 11, 12, 14, 10, 11, 14, 11, 16, 8, and 13. What are the measures of center of the data set?

Find the mean.

$$\begin{aligned}\text{mean} &= \frac{\text{sum of the values}}{\text{number of values}} \\ &= \frac{11 + 12 + 14 + 10 + 11 + 14 + 11 + 16 + 8 + 13}{10} \\ &= \frac{120}{10} = 12\end{aligned}$$

Find the median.

There is an even number of values, so the median is the mean of the two middle values.

8, 10, 11, 11, **11, 12**, 13, 14, 14, 16

$$\text{median} = \frac{11 + 12}{2} = 11.5$$

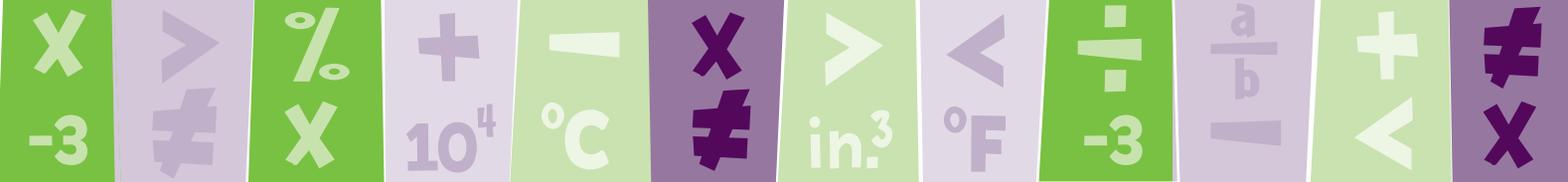
Find the mode.

The number 11 appears three times in the data set, more than any other number.

The data set has a mean of 12, a median of 11.5, and a mode of 11.

## APPLY

A student received an 84, an 80, an 80, and a 92 on four tests. Choose the measure of center that would give the student the highest average. Explain.



## Guided Practice

Billy surveys 13 students in his class by asking, “How many pets do you have?”  
The results are shown below.

7, 0, 3, 2, 0, 0, 12, 1, 0, 4, 1, 2, 7

Determine whether Billy’s question is statistical. Then, determine the measures of center from his data.

**Step 1** Determine whether the question is a statistical question.

Will the answers to Billy’s question vary?

\_\_\_\_\_

So, Billy’s question \_\_\_\_\_ a statistical question.

### REMEMBER

A statistical question will provide a variety of answers.

**Step 2** Find the mean of the data set.

Find the sum of all the values. \_\_\_\_\_

Count the number of values in the data set. \_\_\_\_\_

Divide the sum by the number of values: \_\_\_\_\_ ÷ \_\_\_\_\_ = \_\_\_\_\_

### THINK

The number 0 does not change a sum, but it must be counted in the number of values.

**Step 3** Find the mode of the data set.

The number 0 appears \_\_\_\_\_ times.      The number 1 appears \_\_\_\_\_ times.

The number 2 appears \_\_\_\_\_ times.      The number 7 appears \_\_\_\_\_ times.

The number(s) that appears most often is \_\_\_\_\_.

### REMEMBER

A data set can have no modes, 1 mode, or more than 1 mode.

**Step 4** Find the median of the data set.

List the values in the data set in order from least to greatest.

\_\_\_\_\_

The number in the middle of the data set is \_\_\_\_\_.

The measures of center are mean = \_\_\_\_\_, median = \_\_\_\_\_, and mode = \_\_\_\_\_.

## Independent Practice

- Describe how you can find the mean of a data set.

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Determine whether each question is a statistical question or not. Write *yes* or *no*.

- In what year was the U.S. voting age reduced to 18? \_\_\_\_\_
- What were the ages of voters in the last election? \_\_\_\_\_
- What scores did the students in Ms. Fried's class get on the last quiz? \_\_\_\_\_
- How many students are in Ms. Fried's class? \_\_\_\_\_

**Solve.**

- The table below shows how many pieces of mail Tomas received one week.

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Pieces of Mail Received	6	6	11	12	10

What is the mean of the numbers of pieces of mail that Tomas received?

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What is the median of the numbers of pieces of mail that Tomas received?

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What is the mode of the numbers of pieces of mail that Tomas received?

---





Find the mean, median, and mode of each data set.

7. 14, 17, 15, 23, 10, 23

mean: \_\_\_\_\_

median: \_\_\_\_\_

mode: \_\_\_\_\_

8. 81, 99, 89, 91, 85, 95

mean: \_\_\_\_\_

median: \_\_\_\_\_

mode: \_\_\_\_\_

9. 45, 55, 30, 101, 90, 30, 90

mean: \_\_\_\_\_

median: \_\_\_\_\_

mode: \_\_\_\_\_

10. 10, 19, 49, 20, 0, 38, 83, 97

mean: \_\_\_\_\_

median: \_\_\_\_\_

mode: \_\_\_\_\_

**Solve.**

11. A nature park has six hiking trails. The following table shows the names of the trails and their lengths, in km.

Trail	Northern	Woodsy	Belleview	Grand	Falls	Marathon
Length (in km)	7	4	11	17	7	26

What is the mean length of a hiking trail in the nature park? \_\_\_\_\_

What is the median length of a hiking trail in the nature park? \_\_\_\_\_

What is the mode length of a hiking trail in the nature park? \_\_\_\_\_