# Teacher's Manual

# Support Coach™

## Algebra I

# Contents

LESSON 1

# Interpreting Expressions

| | | CONCEPTS AND SKILLS | VOCABULARY |
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| FOUNDATIONAL UNDERSTANDING | **FUEL UP**<br>**Parts of Expressions**<br>Student Edition pp. 2–4 | Identify the parts of expressions.<br>Write expressions from verbal descriptions, and write verbal descriptions from expressions. | • **expression**<br>• **sum**<br>• **difference**<br>• **product**<br>• **quotient**<br>• **factor**<br>• **coefficient**<br>• **variable** |
| | **SYSTEMS CHECK**<br>**Equivalent Expressions**<br>Student Edition pp. 5–7 | Describe the meanings of equivalent expressions in a real-world situation.<br>Identify equivalent expressions as expressions that represent the same quantity. | • **equivalent expression** |
| ON-LEVEL TARGET | **LAUNCH**<br>**Interpreting Expressions**<br>Student Edition pp. 8–12 | Interpret expressions and their component parts in terms of a context. | |

## MISCONCEPTIONS AND COMMON ERRORS

### Misconceptions about the Order of Terms

When writing an expression from a description, many students always write the variable first, even when that order does not match the meaning. For example, students may incorrectly write the expression "$x$ less than 3" as $x - 3$ instead of $3 - x$.

***What to Look For***

*Sample Problem* Have students solve this problem:
Which expression is equivalent to "$x$ less than 5?"

A) $x < 5$
B) $x - 5$
C) $5 - x$

**Analysis**

Answer A: students are reading the phrase as an inequality, not as an expression.

Answer B: students are writing the terms with the variable first, or in the order given in the verbal description, without understanding the meaning of the expression.

Answer C: correct answer.

**Correcting the Misconception**

Students need to understand the meaning of a verbal description to correctly write the algebraic expression. Try this activity that involves building understanding through students' understanding of the order of numbers.

Step 1: Ask students what "less than 5" means.

*Say: How do you know what a number less than 5 is? (Numbers to the left of 5 on the number line are less than 5.) How can you find a number that is less than 5? (Subtract a positive number from 5.)*

Step 2: Ask students what "3 less than 5" means.

*Say: What number is 3 less than 5? (2) How did you find this number? (subtract 3 from 5, or count back 3 from 5) What expression gives this number? (5 − 3)*

Step 3: Ask students what "$x$ less than 5" means.

*Say: What does it mean that a number is x less than 5? (start at 5 and go back or subtract x) How could you use your expression for "3 less than 5" as a model to find the answer to this question? (replace 3 with x) What expression means "x less than 5?" (5 − x)*

Finally, check student understanding with similar expressions, such as "2 less than $y$" or "$x$ less than $-3$."

### Common Errors in Interpreting Expressions

Some students incorrectly apply the order of operations on expressions involving exponents.

***What to Look For***

*Sample Problem* Look for this type of error in student solutions:
What is the meaning of $(3w)(2x)^3$?

$$(3w)(2x)^3 = (3w \cdot 2x)^3$$
$$= (6wx)^3$$

**Analysis**

Student thought that the exponent applies to the product of $3w$ and $2x$.

**Correcting the Common Error**

Ask students to write the expression showing operations that are not explicitly written, and then have them describe the expression.

The expression above would be written as:

$$(3w) \cdot (2x)^3$$

This expression is the product of $3w$ and $(2x)^3$. Point out that the order of operations tells them to apply exponents before multiplying, so the exponent 3 applies only to $2x$.

# FUEL UP Parts of Expressions

## BUILD BACKGROUND

- MP6 Have students discuss situations when they use the words *sum, difference, product,* or *quotient,* and give examples using each word.
- Tell students they will translate between verbal phrases and mathematical expressions.

## INTRODUCE AND MODEL

**Introduce Concepts and Vocabulary** Guide students through the diagram showing samples of expressions. Make sure they can identify the sums, differences, products, and quotients in the expressions. Also, make sure they can correctly identify coefficients, variables, and factors.

**DISCUSS** MP8 Possible answer: The first is a sum, and one of the terms being added is a product, $4y$. The second is a product, and one of the factors being multiplied is a sum, $(4 + y)$.

**Model Application**

**DO** A Guide students through the steps of writing the expression. Check that $x$ is the dividend and 2 is the divisor in the expression using a division symbol, and that $x$ is in the numerator and 2 is in the denominator in the expression written as a fraction.

**Scaffold Understanding** Some students may not understand the relationship between fractions and division. Support this understanding with a simple example, such as two friends sharing one pizza: each friend gets $1 \div 2 = \frac{1}{2}$ pizza.

**Answer Key**

1. division
2. $x$; 2
3. $x \div 2$; fraction; $\frac{x}{2}$

**DO** B Discuss the order in which subtraction and multiplication are performed in the expression.

**Scaffold Understanding** Some students may name $a$ as the second factor in step 1. To help them understand that the difference is the second factor, contrast the given expression to the expression "the product of 3 and 5". Ask students to identify the factors in this expression, and circle them. Then write the given expression on the board, and compare the structures of the two expressions.

**Answer Key**

1. 3; the difference of $a$ and 7
2. subtraction; $a - 7$
3. $3(a - 7)$

**DISCUSS** MP8 Possible answer: When two operations are used, you should put the expression using the second operation inside parentheses.

**DO** C Unlike the other operations, multiplication can be represented without a symbol. Review expressions such as $2x$, and ensure that students understand that that multiplication is implied.

**Scaffold Understanding** Help students understand that when an expression includes two operations, the last one performed is the first one named. If you were evaluating this expression for a specific value of $y$, you would add 3 and $y$, then multiply the sum by 5. Since you perform multiplication last, the verbal description will specify "product" first.

**Answer Key**

1. multiplied by
2. sum; 3; $y$
3. product; 5; sum; 3; $y$

**DISCUSS** MP7 Possible answer: The order of the factors changed, so the verbal description would be "the product of the sum of 3 and $y$, and 5."

**SPOTLIGHT ON MATHEMATICAL LANGUAGE**

**MP6 Attend to Precision**

Support students in using mathematical language as they work:

What are two numbers that are multiplied called? What is the result of the multiplication called?

What are two numbers that are added called? What is the result of the addition called?

What is the result of division called?

If you reduce a number by another number, what is the result called?

**ENGLISH LANGUAGE LEARNERS**

MP7 Help students identify the parts of mathematical phrases involving sums, differences, products, and quotients. Write several phrases on the board, such as "the sum of $x$ and 3," and "the product of 2 and $y$." Have students underline "sum of" and "and" in the first expression, and "product of" and "and" in the second expression. Then have them circle the addends $x$ and 3 in the first expression and the factors 2 and $y$ in the second expression. Repeat with more complex expressions, such as "the sum of 5 and the product of 3 and $x$."

## PRACTICE AND ASSESS

- Ask students to complete practice items 1–18 on page 4 independently or in pairs. Monitor ongoing work.

  **15–18** Remind students to read the **REMEMBER**. Tell them that the first operation they name is the "big picture" operation. The other operation(s) give details about what is being added, multiplied, etc.

- When students have completed the Practice, assign the Lesson 1 Fuel Up Quiz.

**Answer Key**

1. B
2. C
3. $9 + b$
4. $w - 3$
5. $4x$
6. $\frac{b}{2}$
7. $4 + 3t$
8. $2x - 9$
9. $(11 + c)(b + 1)$
10. $1 + \frac{z}{5}$
11. the difference of $q$ and 2
12. the product of 2 and $x$
13. the sum of $g$ and $h$
14. the quotient of 3 and $n$
15. the product of 4 and the difference of 1 and $x$
16. the quotient of the sum of 2 and $b$, and 5
17. the difference of the product of 5 and $d$ and the quotient of $c$ and 2
18. the product of the sum of 2 and $v$ and the sum of 3 and $w$

**COMMON ERRORS**

When writing expressions using subtraction where the verbal phrase includes "less than," students may write the minuend and subtrahend in the wrong order, following the order in which the terms are given. For example, they may write "3 less than $w$" as $3 - w$. Stress that they should think about the meaning of the words. Then ask what the value of 3 less than $w$ should be if $w = 15$, and ask what expression produces this value.

# SYSTEMS CHECK Equivalent Expressions

## BUILD BACKGROUND

- Discuss situations where numbers can be understood in different ways. For example, if one class has 5 more students than another, you could say the total number of students is the sum of the two classes, $x + (x + 5)$. Or, you could say that it is 5 more than twice the number in the smaller class, $2x + 5$.
- **MP4** Have students discuss additional examples of situations where the same value can be written and explained in different ways.
- Tell students they will describe the meanings of real-world equivalent expressions, and identify equivalent expressions.

**SPOTLIGHT ON MATHEMATICAL PRACTICES**

**MP1 Make Sense of Problems**

Develop students' understanding of equivalent expressions by challenging them to write as many expressions as they can that are equivalent to 10. Then repeat with more complex expressions, such as $5x$ and $4x + 3$.

## INTRODUCE AND MODEL

**Introduce Concepts and Vocabulary** Explain that an expression may or may not include variable(s). If expressions do contain a variable, then they are equivalent only if they evaluate to the same value for any value of the variable. The expressions $5x + 1$ and $6x + 1$ are both equal to 1 when $x = 0$, but for any other value of $x$, the expressions do not have the same value. Therefore, they are not equivalent.

**DISCUSS** MP1 Possible answer: Yes, just like any number can be written in different ways, every expression can be written in different ways.

**Model Application**

**DO** A If students have difficulty understanding that the expressions are equivalent, draw a diagram of a different rectangle with whole number length and width. Ask students for the perimeter of the rectangle, and show how each expression evaluates to that value for that length and width.

**Scaffold Understanding** Encourage students to refer back to the diagram as they work through each expression to explain its meaning.

**Answer Key**

1. 4; side; the sum of the lengths of the 4 sides
2. products; twice the width; twice the length; sum; twice the width; twice the length
3. the sum of the width and length; twice the sum of the width and length

**DO** B Discuss with students that although there are infinitely many expressions that are equivalent to $p - 0.3p$, not all equivalent expressions allow you to see the real-world situation represented by the expression from a different perspective.

**Scaffold Understanding** Remind students that any number times 1 is that number. So, if a variable does not have a coefficient written in front of it, you can write it with a coefficient of 1.

**Answer Key**

1. the original price; 30% of the original price; the original price minus 30% of the original price
2. 1; 0.7; $0.7p$
3. $0.7p$; 0.7; 70

**DO** C Guide students to assess whether two expressions correctly represent a quantity. If they do, then they are equivalent expressions.

**Scaffold Understanding** Help students focus on the story that each expression gives. The first expression shows that each sister pays one fourth of the total cost. The second expression shows that each sister pays for half of one ticket plus $25 for the gift card.

**COMMON ERRORS**

Students may incorrectly factor $p$ from the terms in step 2. To help them understand this step, give a more concrete example, such beginning with 5 groups of 4, and taking away 2 groups of 4. Once students understand that the result is 3 groups of 4, work towards more abstract examples using a variable as the group size.

**Answer Key**

1. $2t$; 100; $2t + 100$
2. 4; one fourth; correct
3. 4; $2t$; $0.5t$; 100; 25; $0.5t + 25$; correct

**DISCUSS** **MP1** Possible answer: The first expression; it shows the total cost and that the cost is divided among 4 people.

## PRACTICE AND ASSESS

- Ask students to complete practice items 1–11 on page 7 independently or in pairs. Monitor ongoing work.

  1 Remind students to read the **HINT**. Tell students that it is important to try to think about the value represented by an expression from different perspectives.

- When students have completed the Practice, assign the Lesson 1 Systems Check Quiz.

**Answer Key**

1. $ab + 5a$: the area of the rectangle is the combined area of the two smaller rectangles, which have area $ab$ and $5a$; $a(b + 5)$: the area of the rectangle is length times width, and the two dimensions are $a$ and $(b + 5)$
2. $60(a + b)$: Ellie drove 60 miles per hour for a total time of $a + b$ hours, so her total distance was $60(a + b)$; $60a + 60b$: Ellie drove $60a$ miles before lunch and $60b$ miles after lunch for a total distance of $60a + 60b$.
3. Possible answer: $1.08p$
4. Possible answer: $5(g - 2)$
5. Possible answer: $5c$
6. Possible answer: $6n$
7. no; each boy only pays for half of the service charge, or $4
8. yes; this expression shows the total charge for 2 tickets and the service charge divided by 4
9. yes; this expression shows that the cost of 2 tickets plus the extra charges ($5 for each ticket at $8 for the order) are divided by 2
10. yes; this expression shows half of each charge: half of each ticket, half of each facility charge, and half of the service fee
11. no; this expression does not include each boy's portion of the service charge

**SPOTLIGHT ON MATHEMATICAL PRACTICES**

**MP7 Look for Structure**

Discuss with students how they can use the structure of an expression to help interpret it. For example, in item 2, $60(a + b)$ is 60 times a number, and since 60 represents the speed in miles per hour, $a + b$ likely represents time in hours. Ask students to extend this type of thinking to other expressions, and ask them to explain their thinking.

# LAUNCH Interpreting Expressions

## BUILD BACKGROUND

- Talk to students about when they might need to interpret expressions in real life. For example, most spreadsheet programs allow the use of formulas, which are expressions. Interpreting the expressions allows you to check whether the formula makes sense.
- MP1 Remind students of problem-solving strategies they can use, such as starting with smaller expressions within the expression, or testing the expression for specific values of the variable to help understand the value it gives.
- Tell students they will work on interpreting expressions that are given in real-world situations.

## INTRODUCE AND MODEL

**Introduce Concepts and Vocabulary** Guide students through the example expression and its meaning. Lead students to see that while each part has a specific meaning, the overall meaning of the expression combines those meanings in a unique way.

DISCUSS MP1 Possible answer: Multiplication shows combining groups, such as 8 groups of *m*. Division shows making equal sized groups. Addition and subtraction show combining and taking away.

## LESSON LINK

Connect to Foundational Understanding Skills learned in the Fuel Up and Systems Check are referenced in the Lesson Link. Explain to students that describing an expression with words and analyzing equivalent expressions are two tools they can use to help interpret expressions.

**Model Application**

DO A Students will find it easier to interpret the expression if they are familiar with the situation. Discuss the scenario with students, including different possible values for *b* and *p*.

**Scaffold Understanding** Students may have little experience with expressions containing two variables. Support students by listing each variable and its meaning on the board for reference.

**Answer Key**

1. 24; pens in a box; *b*; pens
2. the number of pens sold
3. pens; remain

DO B Students are told the meaning of an expression, and use the meaning of the expression and the terms in the expression to identify additional information about the situation.

**Scaffold Understanding** Students may struggle to understand the steps to determine Henry's regular hourly pay from the 800 term. Stress that he must be paid for every hour that he works, and the second term covers his hours over 40 hours. Therefore, $800 must be his pay for 40 hours, and you can divide by 40 to find the hourly rate.

**Answer Key**

1. 800; 800
2. product; factors; $(h - 40)$; overtime
3. 30; 40; 40; 20

**DISCUSS** MP7 Possible answer: The first term would have to include his pay for 50 hours. The second term would include the factor $(h - 50)$.

**DO** C After completing the example, challenge students to come up with another description that matches the expression. Have students discuss the suggestions and verify that the situation is modeled by the expression.

**Scaffold Understanding** Ensure that students understand why choice A needs to include the term $2(x - 12)$, but choice B includes $2x - 12$.

Answer Key

1. 12; $2(x - 12)$; $2(x - 12) + 3$; incorrect
2. $x - 12$; $2(x - 12) + 3x$; correct
3. B

**DO** D The final step in this example does not have just one correct answer. Encourage students to use their real-world knowledge to think about why the total amount paid might be 115% of the total cost of the items.

**Scaffold Understanding** Because the second factor is long, students may not recognize that the expression is the product of two factors. To help them see the structure, circle each set of parentheses and its contents with a different color.

**Answer Key**

1. 2; $(2s + 2(s - 3) + (s + 1))$
2. a drink; fruit; 2 sandwiches; 2 drinks; 1 fruit
3. 1.15; 2 sandwiches, 2 drinks, and 1 fruit; possible answer: the cost of these items including a 15% tip

**DISCUSS** MP1 Possible answer: The second factor is a complex expression, but the last operation performed is multiplying 1.15 by the value of this expression, so the expression is a product.

## SUPPORT INDEPENDENT PRACTICE

- Ask students to complete practice items 1–16 on pages 11–12 independently or in pairs. Monitor ongoing work.

  2 Remind students to read the **HINT**. Tell students that when adding values, the units must be the same.

**ENGLISH LANGUAGE LEARNERS**

MP7 Ensure that students can identify the terms in an expression. Have students copy several expressions, such as $5x + 9$, $4(x - 1) + 3x$, and $\frac{15x}{2} + 3$, and have them highlight or underline each term. Discuss their thinking in identifying the terms.

**COMMON ERRORS**

Students may select a description of an expression because they see all the numbers that are given in the expression. Encourage them to test the expression with specific values for the variable to determine the value of the expression, then determine whether that value matches the value described.

**Answer Key**

1. the number of minutes she practices piano and guitar each day
2. the number of minutes she practices piano and guitar each week
3. the total number of minutes she practices each week
4. the amount of food the large dogs eat at each meal
5. the amount of food the small dogs eat at each meal
6. the amount of food all his dogs eat at one meal
7. the total amount of food his dogs eat in a day
8. C
9. A
10. the first 4 caps cost $4 each, and the price of more caps is $3
11. B
12. C
13. the height of an apartment building that has $x$ floors
14. the total number of people going on the field trip in 5 buses if 7 teachers go in separate cars
15. the cost, after a 20% discount is applied, of 2 pairs of jeans and 3 T-shirts
16. Tina's average points scored over the two games

## ERROR ANALYSIS

**Summary/Objectives**

Many students apply the order of operations incorrectly when analyzing expressions, and therefore interpret the expressions incorrectly. Some students may find it helpful to write in a multiplication symbol where it is implied. Also, listing the order of operations where it can be easily referenced may help students to interpret expressions correctly.

**Answer Key**

Calvin incorrectly stated that $4 \div n$ is the quotient of $n$ and 4. It is the quotient of 4 and $n$. Also, the exponent only applies to the factor 3, not the product $(4 \div n)3$.

$4 \div n$ is the quotient of 4 and $n$.

$3^2$ is 3 squared.

$(4 \div n)3^2$ is the product of the quotient of 4 and $n$, and 3 squared.

## ASSESS

- Use the table below to observe whether students accurately interpret expressions.
- When all students are ready, assign the Lesson 1 LAUNCH Quiz.

| Observation | Action |
|---|---|
| Errors in identifying the parts of expressions or writing expressions from verbal descriptions. | Review the definitions of sum, difference, product, quotient, factor, variable, and coefficient. |
| Errors in identifying equivalent expressions. | Have students explain the meaning of simple expressions. Have them test expressions using several values of the variable to check if they might be equivalent. |
| Interprets expressions correctly. | Assign the Lesson 1 LAUNCH Quiz |